Module 7

Transformer

Lesson 24

Practical Transformer

Contents

24.1 Goals of the lesson

In practice no transformer is ideal. In this lesson we shall add realities into an ideal transformer for correct representation of a practical transformer. In a practical transformer, core material will have (i) finite value of μ_r , (ii) winding resistances, (iii) leakage fluxes and (iv) core loss. One of the major goals of this lesson is to explain how the effects of these can be taken into account to represent a practical transformer. It will be shown that a practical transformer can be considered to be an ideal transformer plus some appropriate resistances and reactances connected to it to take into account the effects of items (i) to (iv) listed above.

Next goal of course will be to obtain exact and approximate equivalent circuit along with phasor diagram.

Key words : leakage reactances, magnetizing reactance, no load current.

After going through this section students will be able to answer the following questions.

- How does the effect of magnetizing current is taken into account?
- How does the effect of core loss is taken into account?
- How does the effect of leakage fluxes are taken into account?
- How does the effect of winding resistances are taken into account?
- Comment the variation of core loss from no load to full load condition.
- Draw the exact and approximate equivalent circuits referred to primary side.
- Draw the exact and approximate equivalent circuits referred to secondary side.
- Draw the complete phasor diagram of the transformer showing flux, primary & secondary induced voltages, primary & secondary terminal voltages and primary & secondary currents.

24.2 Practical transformer

A practical transformer will differ from an ideal transformer in many ways. For example the core material will have finite permeability, there will be eddy current and hysteresis losses taking place in the core, there will be leakage fluxes, and finite winding resistances. We shall gradually bring the realities one by one and modify the ideal transformer to represent those factors.

Consider a transformer which requires a finite magnetizing current for establishing flux in the core. In that case, the transformer will draw this current I_m even under no load condition. The level of flux in the core is decided by the voltage, frequency and number of turns of the primary and does not depend upon the nature of the core material used which is apparent from the following equation:

$$\phi_{\max} = \frac{V_1}{\sqrt{2}\pi f N_1}$$

Hence maximum value of flux density B_{max} is known from $B_{max} = \frac{\phi_{\max}}{A_i}$, where A_i is the net cross sectional area of the core. Now H_{max} is obtained from the B – H curve of the material. But we know $H_{max} = \frac{N_1 I_{m\max}}{l_i}$, where I_{mmax} is the maximum value of the magnetizing current. So rms value of the magnetizing current will be $I_{\mathrm{m}} = \frac{I_{m\max}}{\sqrt{2}}$. Thus we find that the amount of magnetizing current drawn will be different for different core material although applied voltage, frequency and number of turns are same. Under no load condition the required amount of flux will be produced by the mmf $N_1 I_m$. In fact this amount of mmf must exist in the core of the transformer all the time, independent of the degree of loading.

Whenever secondary delivers a current I_2, The primary has to reacts by drawing extra current I'_2 (called reflected current) such that $I'_2 N_1 = I_2 N_2$ and is to be satisfied at every instant. Which means that if at any instant i_2 is leaving the dot terminal of secondary, i_2' will be drawn from the dot terminal of the primary. It can be easily shown that under this condition, these two mmfs (i.e, $N_2 i_2$ and $i_2' N_1$) will act in opposition as shown in figure 24.1. If these two mmfs also happen to be numerically equal, there can not be any flux produced in the core, due to the effect of actual secondary current I_2 and the corresponding reflected current I_2'

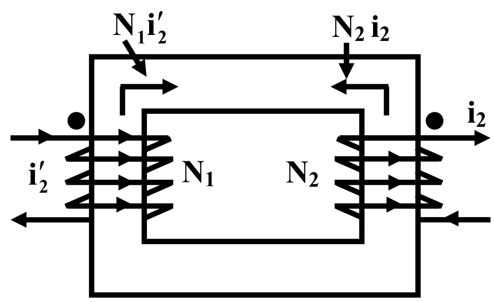
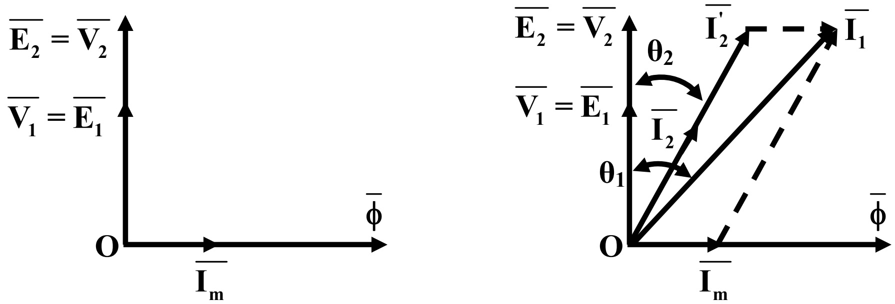


Figure 24.1: MMf directions by I_2 and I_2'

The net mmf therefore, acting in the magnetic circuit is once again $I_m N_1$ as mmfs $I_2' N_1$ and $I_2 N_2$ cancel each other. All these happens, because KVL is to be satisfied in the primary demanding ϕ_{max} to remain same, no matter what is the status (i., open circuited or loaded) of the secondary. To create ϕ_{max}, mmf necessary is $N_1 I_m$. Thus, net mmf provided by the two coils together must always be $N_1 I_{\mathrm{m}}$ – under no load as under load condition. Better core material is used to make I_m smaller in a well designed transformer.

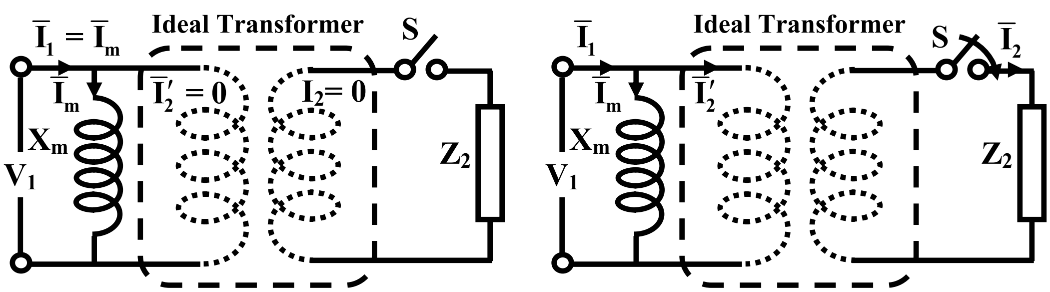
Keeping the above facts in mind, we are now in a position to draw phasor diagram of the transformer and also to suggest modification necessary to an ideal transformer to take magnetizing current $\overline{I}_m$ into account. Consider first, the no load operation. We first draw the $\overline{\phi}_{\max}$ phasor. Since the core is not ideal, a finite magnetizing current $\overline{I}_m$ will be drawn from supply and it will be in phase with the flux phasor as shown in figure 24.2(a). The induced voltages in primary $\overline{E}_1$ and secondary $\overline{E}_2$ are drawn 90º ahead (as explained earlier following convention 2). Since winding resistances and the leakage flux are still neglected, terminal voltages $\overline{V}_1$ and $\overline{V}_2$ will be same as $\overline{E}_1$ and $\overline{E}_2$ respectively.



(a) No load condition **(b) Loaded condition**

Figure 24.2: Phasor Diagram with magnetising current taken into account.

If you compare this no load phasor diagram with the no load phasor diagram of the ideal transformer, the only difference is the absence of $\overline{I}_m$ in the ideal transformer. Noting that $\overline{I}_m$ lags $\overline{V}_1$ by 90º and the magnetizing current has to supplied for all loading conditions, common sense prompts us to connect a reactance X_m, called the magnetizing reactance across the primary of an ideal transformer as shown in figure 24.3(a). Thus the transformer having a finite magnetizing current can be modeled or represented by an ideal transformer with a fixed magnetizing reactance X_m connected across the primary. With S opened in figure 24.3(a), the current drawn from the supply is $\overline{I}_1 = \overline{I}_m$ since there is no reflected current in the primary of the ideal transformer. However, with S closed there will be $\overline{I}_2$, hence reflected current $\overline{I}_2' = \overline{I}_2 / a$ will appear in the primary of the ideal transformer. So current drawn from the supply will be $\overline{I}_1 = \overline{I}_m + \overline{I}_2'$.



(a) with secondary opened **(a) with load in secondary**

Figure 24.3: Magnetising reactance to take I_m into account.

This model figure 24.3 correctly represents the phasor diagram of figure 24.2. As can be seen from the phasor diagram, the input power factor angle θ_1 will differ from the load power factor θ_2 in fact power factor will be slightly poorer (since $\theta_2 > \theta_1$).

Since we already know how to draw the equivalent circuit of an ideal transformer, so same rules of transferring impedances, voltages and currents from one side to the other side can be revoked here because a portion of the model has an ideal transformer. The equivalent circuits of the transformer having finite magnetizing current referring to primary and secondary side are shown respectively in figures 24.4(a) and (b).

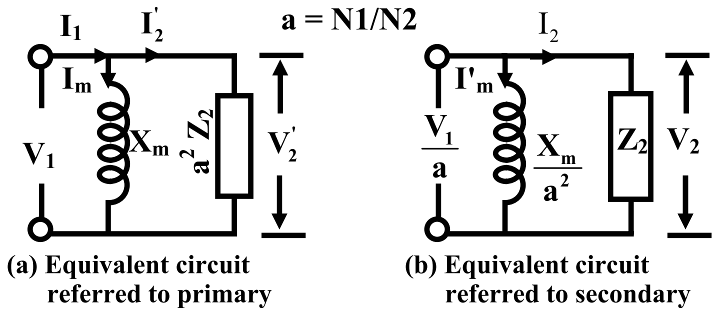


Figure 24.4: Equivalent circuits with X_m

24.2.1 Core loss

A transformer core is subjected to an alternating *time varying field* causing eddy current and hysteresis losses to occur inside the core of the transformer. The sum of these two losses is known as *core loss* of the transformer. A detail discussion on these two losses has been given in Lesson 22.

Eddy current loss is essentially I^2 R loss occurring inside the core. The current is caused by the induced voltage in any conceivable closed path due to time varying field. Obviously to reduce eddy current loss in a material we have to use very thin plates instead of using solid block of material which will ensure very less number of available eddy paths. Eddy current loss per unit volume of the material directly depends upon the *square* of the *frequency, flux density and thickness* of the plate. Also it is inversely proportional to the *resistivity* of the material. The core of the material is constructed using thin plates called lamination. Each plate is given a varnish coating for providing necessary insulation between the plates. *Cold Rolled Grain Oriented*, in short CRGO sheets are used to make transformer core.

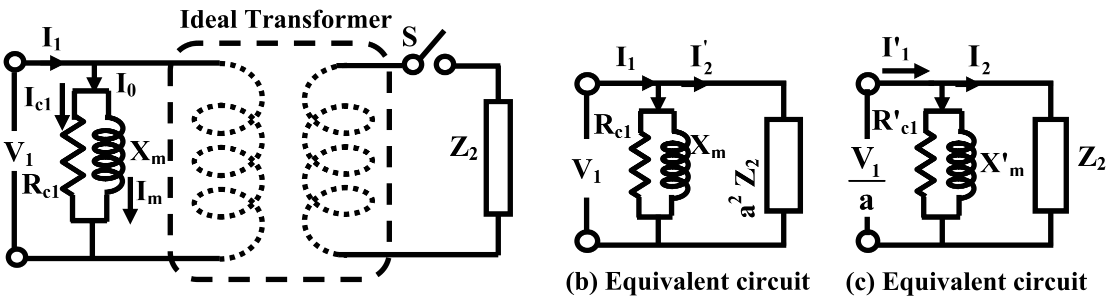
After experimenting with several magnetic materials, Steinmetz proposed the following empirical formula for quick and reasonable estimation of the hysteresis loss of a given material.

$$P_h = k_h B_{\max}^n f$$

The value of n will generally lie between 1.5 to 2.5. Also we know the area enclosed by the hysteresis loop involving B-H characteristic of the core material is a measure of hysteresis loss per cycle.

24.3 Taking core loss into account

The transformer core being subjected to an alternating field at supply frequency will have hysteresis and eddy losses and should be appropriately taken into account in the equivalent circuit. The effect of core loss is manifested by heating of the core and is a real power (or energy) loss. Naturally in the model an external resistance should be shown to take the core loss into account. We recall that in a well designed transformer, the flux density level B_{max} practically remains same from no load to full load condition. Hence magnitude of the core loss will be practically independent of the degree of loading and this loss must be drawn from the supply. To take this into account, a fixed resistance R_{cl} is shown connected in parallel with the magnetizing reactance as shown in the figure 24.5.



(a) R_{cl} for taking core loss into account

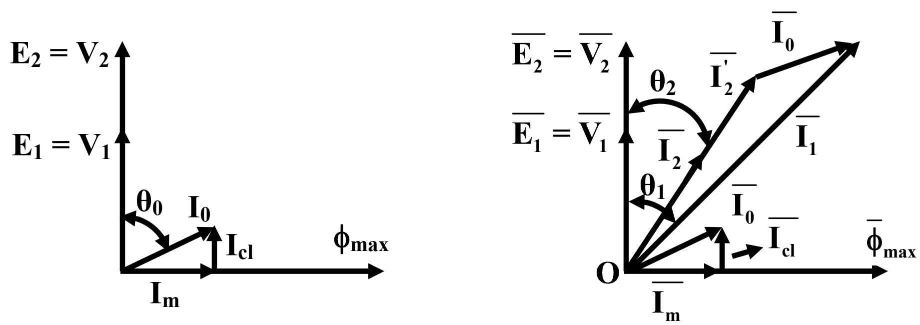
(b) Equivalent circuit referred to primary

(c) Equivalent circuit referred to secondary

Figure 24.5: Equivalent circuit showing core loss and magnetizing current.

It is to be noted that R_{cl} represents the core loss (i.e., sum of hysteresis and eddy losses) and is in parallel with the magnetizing reactance X_m. Thus the no load current drawn from the supply I_o, is not magnetizing current I_m alone, but the sum of I_{cl} and I_m with $\overline{I}_{cl}$ in phase with supply voltage $\overline{V}_1$ and $\overline{I}_m$ lagging by 90º from $\overline{V}_1$. The phasor diagrams for no load and load operations are shown in figures 24.6 (a) and (b).

It may be noted, that no load current I_o is about 2 to 5% of the rated current of a well designed transformer. The reflected current $\overline{I}_2'$ is obviously now to be added vectorially with $\overline{I}_o$ to get the total primary current $\overline{I}_1$ as shown in figure 24.6 (b).



(a) phasor diagram: no load

(b) phasor diagram: with load power factor angle θ_2

Figure 24.6: Phasor Diagram of a transformer having core loss and magnetising current.

24.4 Taking winding resistances and leakage flux into account

The assumption that all the flux produced by the primary links the secondary is far from true. In fact a small portion of the flux only links primary and completes its path mostly through air as shown in the figure 24.7. The total flux produced by the primary is the sum of the mutual and the leakage fluxes. While the mutual flux alone takes part in the energy transfer from the primary to the secondary, the effect of the leakage flux causes additional voltage drop. This drop can be represented by a small reactance drop called the leakage reactance drop. The effect of winding resistances are taken into account by way of small lumped resistances as shown in the figure 24.8.

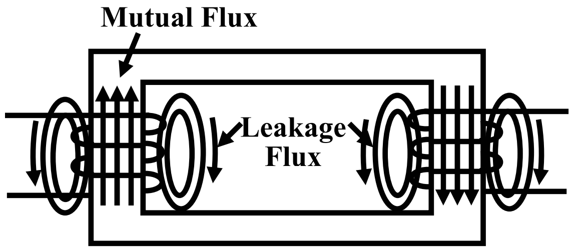
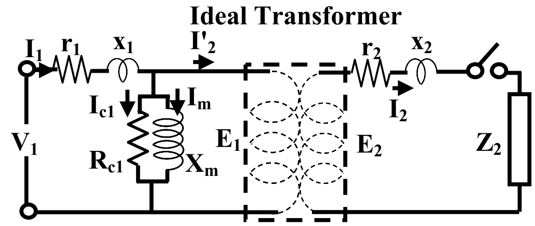


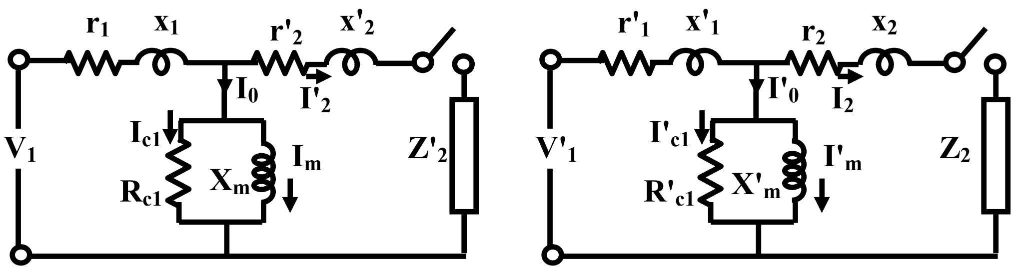
Figure 24.7: Leakage flux and their paths.



Equivalent circuit showing both sides

Figure 24.8: Equivalent circuit of a practical transformer.

The *exact* equivalent circuit can now be drawn with respective to various sides taking all the realities into account. Resistance and leakage reactance drops will be present on both the sides and represented as shown in the figures 24.8 and 24.9. The drops in the leakage impedances will make the terminal voltages different from the induced voltages.



Equivalent circuit referred to primary **Equivalent circuit referred to secondary**

Figure 24.9: Exact Equivalent circuit referred to primary and secondary sides.

It should be noted that the parallel impedance representing core loss and the magnetizing current is much higher than the series leakage impedance of both the sides. Also the no load current I_0 is only about 3 to 5% of the rated current. While the use of exact equivalent circuit will give us exact modeling of the practical transformer, but it suffers from computational burden. The basic voltage equations in the primary and in the secondary based on the exact equivalent circuit looks like:

$$\overline{V_1} = \overline{E_1} + \overline{I_1}\, r_1 + j\overline{I_1}\, x_1$$
$$\overline{V_2} = \overline{E_2} - \overline{I_2}\, r_2 - j\overline{I_2}\, x_2$$

It is seen that if the parallel branch of R_{cl} and X_m are brought forward just right across the supply, computationally it becomes much more easier to predict the performance of the transformer sacrificing of course a little bit of accuracy which hardly matters to an engineer. It is this *approximate equivalent circuit* which is widely used to analyse a practical power/distribution transformer and such an equivalent circuit is shown in the figure 24.11.

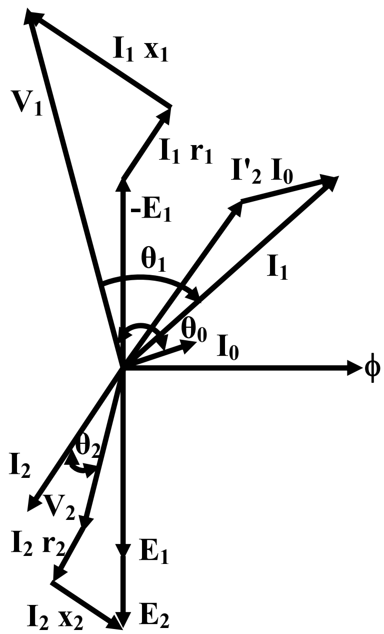


Figure 24.10: Phasor diagram of the transformer supplying lagging power factor load.

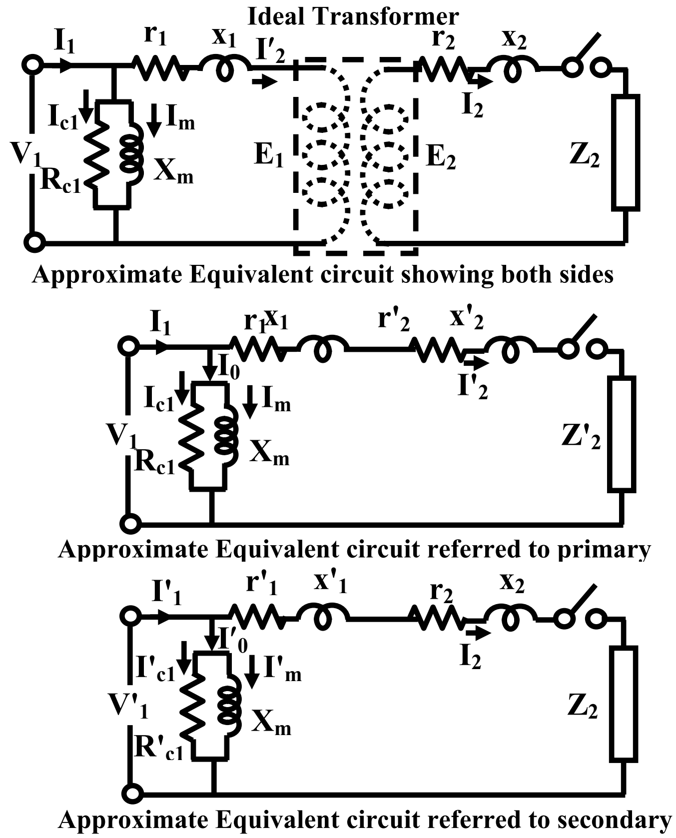


Figure 24.11: Approximate equivalent circuit.

The exact phasor diagram of the transformer can now be drawn by drawing the flux phasor first and then applying the KVL equations in the primary and in the secondary.

The phasor diagram of the transformer when it supplies a lagging power factor load is shown in the figure 24.10. It is clearly seen that the difference in the terminal and the induced voltage of both the sides are nothing but the leakage impedance drops of the respective sides. Also note that in the approximate equivalent circuit, the leakage impedance of a particular side is in series with the reflected leakage impedance of the other side. The sum of these leakage impedances are called *equivalent leakage impedance referred* to a particular side.

Equivalent leakage impedance referred to primary $r_{e1} + jx_{e1} = \left(r_1 + a^2 r_2\right) + j\left(x_1 + a^2 x_2\right)$

Equivalent leakage impedance referred to secondary $r_{e2} + jx_{e2} = \left(r_2 + \dfrac{r_1}{a^2}\right) + j\left(x_2 + \dfrac{x_1}{a^2}\right)$

Where, $a = \dfrac{N_1}{N_2}$ the turns ratio.

24.5 A few words about equivalent circuit

Approximate equivalent circuit is widely used to predict the performance of a transformer such as estimating regulation and efficiency. Instead of using the equivalent circuit showing both the sides, it is always advantageous to use the equivalent circuit referred to a particular side and analyse it. Actual quantities of current and voltage of the other side then can be calculated by multiplying or dividing as the case may be with appropriate factors involving turns ratio a.

Transfer of parameters (impedances) and quantities (voltages and currents) from one side to the other should be done carefully. Suppose a transformer has turns ratio a, $a = N_1/N_2 = V_1/V_2$ where, N_1 and N_2 are respectively the primary and secondary turns and V_1 and V_2 are respectively the primary and secondary rated voltages. The rules for transferring parameters and quantities are summarized below.

1. **Transferring impedances from secondary to the primary:**
   If actual impedance on the secondary side be $\bar{Z}_2$, referred to primary side it will become $\bar{Z}_2' = a^2\bar{Z}_2$.

2. **Transferring impedances from primary to the secondary:**
   If actual impedance on the primary side be $\bar{Z}_1$, referred to secondary side it will become $\bar{Z}_1' = \bar{Z}_1 / a^2$.

3. **Transferring voltage from secondary to the primary:**
   If actual voltage on the secondary side be $\bar{V}_2$, referred to primary side it will become $\bar{V}_2' = a\bar{V}_2$.

4. **Transferring voltage from primary to the secondary:**
   If actual voltage on the primary side be $\bar{V}_1$, referred to secondary side it will become $\bar{V}_1' = \bar{V}_1 / a$.

5. **Transferring current from secondary to the primary:**
   If actual current on the secondary side be $\bar{I}_2$, referred to primary side it will become $\bar{I}_2' = \bar{I}_2 / a$.

6. **Transferring current from primary to the secondary:**
   If actual current on the primary side be $\bar{I}_1$, referred to secondary side it will become $\bar{I}_1' = a\bar{I}_1$.

In spite of all these, gross mistakes in calculating the transferred values can be identified by remembering the following facts.

1. A current referred to LV side, will be higher compared to its value referred to HV side.
2. A voltage referred to LV side, will be lower compared to its value referred to HV side.
3. An impedance referred to LV side, will be lower compared to its value referred to HV side.

24.6 Tick the correct answer

1. If the applied voltage of a transformer is increased by 50%, while the frequency is reduced to 50%, the core flux density will become

   [A] 3 times [B] $\frac{3}{4}$ times [C] $\frac{1}{3}$ [D] same

2. For a 10 kVA, 220 V / 110 V, 50 Hz single phase transformer, a good guess value of no load current from LV side is:

   (A) about 1 A (B) about 8 A (C) about 10 A (D) about 4.5 A

3. For a 10 kVA, 220 V / 110 V, 50 Hz single phase transformer, a good guess value of no load current from HV side is:

   (A) about 2.25 A (B) about 4 A (C) about 0.5 A (D) about 5 A

4. The consistent values of equivalent leakage impedance of a transformer, referred to HV and referred to LV side are respectively:

   (A) $0.4 + j0.6\Omega$ and $0.016 + j0.024\Omega$
   (B) $0.2 + j0.3\Omega$ and $0.008 + j0.03\Omega$
   (C) $0.016 + j0.024\Omega$ and $0.4 + j0.6\Omega$
   (D) $0.008 + j0.3\Omega$ and $0.016 + j0.024\Omega$

5. A 200 V / 100 V, 50 Hz transformer has magnetizing reactance $X_m = 400\Omega$ and resistance representing core loss $R_{cl} = 250\Omega$ both values referring to HV side. The value of the no load current and no load power factor referred to HV side are respectively:

   (A) 2.41 A and 0,8 lag (B) 1.79 A and 0.45 lag
   (C) 3.26 A and 0.2 lag (D) 4.50 A and 0.01 lag

6. The no load current drawn by a single phase transformer is found to be $i_0 = 2 \cos \omega t$ A when supplied from 440 cos ωt Volts. The magnetizing reactance and the resistance representing core loss are respectively:

   (A) 216.65 Ω and 38.2 Ω (B) 223.57 Ω and 1264 Ω
   (C) 112 Ω and 647 Ω (D) 417.3 Ω and 76.4 Ω

24.7 Solve the problems

1. A 5 kVA, 200 V / 100 V, 50Hz single phase transformer has the following parameters:

HV winding Resistance	=	0.025 Ω
HV winding leakage reactance	=	0.25 Ω
LV winding Resistance	=	0.005 Ω
LV winding leakage reactance	=	0.05 Ω
Resistance representing core loss in HV side	=	400 Ω
Magnetizing reactance in HV side	=	190 Ω

Draw the equivalent circuit referred to [i] LV side and [ii] HV side and insert all the parameter values.

2. A 10 kVA, 1000 V / 200 V, 50 Hz, single phase transformer has HV and LV side winding resistances as 1.1 Ω and 0.05 Ω respectively. The leakage reactances of HV and LV sides are respectively 5.2 Ω and 0.15 Ω respectively. Calculate [i] the voltage to be applied to the HV side in order to circulate rated current with LV side shorted, [ii] Also calculate the power factor under this condition.

3. Draw the complete phasor diagrams of a single phase transformer when [i] the load in the secondary is purely resistive and [ii] secondary load power factor is *leading*.